

IN SEARCH OF THE FEEDBACK FUNCTION FOR VARIABLE-INTERVAL SCHEDULES

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Finding a theoretically sound feedback function for variable-interval schedules remains an important unsolved problem. It is important because interval schedules model a significant feature of the world: the dependence of reinforcement on factors beyond the organism's control. The problem remains unsolved because no feedback function yet proposed satisfies all the theoretical and empirical requirements. Previous suggestions that succeed in fitting data fail theoretically because they violate a newly recognized theoretical requirement: The slope of the function must approach or equal 1.0 at the origin. A function is presented that satisfies all requirements but lacks any theoretical justification. This function and two suggested by Prelec and Herrnstein (1978) and Nevin and Baum (1980) are evaluated against several sets of data. All three fitted the data well. The success of the two theoretically incorrect functions raises an empirical puzzle: Low rates of reinforcement are coupled with response rates that seem anomalously high. It remains to be discovered what this reflects about the temporal patterning of operant behavior at low reinforcement rates. A theoretically and empirically correct function derived from basic assumptions about operant behavior also remains to be discovered.

Key words: feedback function, variable-interval schedules, molar relations

The dynamics of behavior may be understood as the outcome of the organism and environment interacting together as a feedback system (Baum, 1973, 1981, 1989; Staddon, 1983). The organism's functional relations (or "O-rules"; Baum, 1973) link environmental events to behavioral output. The environment's feedback functions (or "E-rules"; Baum, 1973) link behavioral output to environmental events (i.e., consequences). Equilibrium results from the interaction between functional relations and feedback functions. Any brief disturbance to the system results in departure from equilibrium, but once the disturbance is gone, the system stabilizes again. Any change in a functional relation (e.g., a shift in deprivation) or in a feedback function (e.g., a change of schedule) results in a new equilibrium (e.g., Myerson & Hale, 1988; Staddon, 1988).

Broadly speaking, any theory of operant behavior must specify two components: what is desirable and what is possible. What is desirable depends on properties of the organism; in the present context, it is set by the organism's functional relations, which might be thought of, for example, with terms like melioration

(Herrnstein & Vaughan, 1980; Vaughan & Herrnstein, 1987) or optimality (Rachlin, 1989; Rachlin, Battalio, Kagel, & Green, 1981; Staddon, 1983). What is possible depends on properties of the environment; in the present context, it is given by the feedback functions imposed by schedules of reinforcement. This paper leaves aside the problem of what is desirable and focuses on the problem of finding an adequate method for specifying what is possible—that is, for specifying environmental constraints in feedback functions.

Before the 1970s, it was common to define and explain schedules of reinforcement by distinguishing between those responses that could produce reinforcers and those that could not. An interval schedule, for example, was said to reinforce only the first response following the timing-out of the interval; all others go unreinforced. If, instead, we think of each schedule as imposing a relation that specifies the consequences, over time, of a performance maintained over time, we arrive at a concrete idea of a feedback function, according to which rate of reinforcement is determined by response rate (Baum, 1973). An interval schedule, for example, imposes a feedback function that is negatively accelerated and approaches a horizontal asymptote (the programmed rate of reinforcement) at high response rates.

This paper has three aims: (a) to explain briefly why specifying the variable-interval (VI) feedback function is an important prob-

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lem, (b) to explain why it is as yet unsolved, and (c) to suggest some possible strategies for solving it.

An Important Problem

The importance of interval schedules is best seen in contrast with ratio schedules. When responses are reinforced according to a ratio schedule, it can be said that reinforcement is completely under the control of responding. A certain number of responses must be completed; nothing else matters. Ratio schedules, however, represent an extreme situation, because the environment often includes other factors that are beyond the control of the responding with which the schedule makes contact. If dialing the telephone produces a busy signal, I can dial the telephone again, but my dialing cannot effect the freeing of the line. Whenever the availability of reinforcement depends on such uncontrollable factors, the dependency between reinforcement and responding shares the character of an interval schedule. The timer in the laboratory substitutes for the uncontrollable factors of the real world.

Because interval-schedule-like situations arise commonly in the behavioral environment, no theory of performance can be complete that fails to offer an account of interval schedules. Seen in the light of the organism-environment feedback system, no theory of performance can be complete without specification of the feedback function for interval schedules.

We need to focus particularly on VI schedules because fixed-interval schedules present the additional complication of an extended postreinforcement pause. The evidence suggests that fixed-interval performance can be divided into this initial pause followed by performance that resembles VI responding in all respects (e.g., Schneider, 1969). Hence, VI schedules can be considered prototypical, and solution of the problem of VI performance is likely to make possible solutions of other situations that share the character of VI schedules (Baum, 1989).

An Unsolved Problem

Several attempts at specifying the VI feedback function have been made. Although some may seem successful, we shall soon see that none has succeeded completely. One reason may be that the characteristics of the required function have never been fully specified.

If r is rate of reinforcement and B is response rate, then the general form of the feedback function is $r = f(B)$, and the question of interest is what properties of the function f can be deduced from the properties of VI schedules.

Two properties are true of any standard ratio or interval schedule of reinforcement. First, the function must pass through the origin. No responding means no reinforcement. Freely delivered reinforcers might be superimposed, but the VI feedback function itself passes through the origin because it reflects response-dependent reinforcement. Second, the function must be continuous over the range of possible response rates. For every response rate that can occur there must be a definite rate of reinforcement, even if that rate of reinforcement is zero.

Besides these general characteristics, the VI feedback function in particular must possess at least two others. First, the function must approach a horizontal asymptote equal to the programmed rate of reinforcement as the response rate increases. This property reflects the limit imposed by the timer (the uncontrollable factor) that defines an interval schedule. When response rate is low, the rate of reinforcement can fall below that which is programmed, but as response rate grows, the obtained rate of reinforcement approaches the programmed rate, because reinforcers are produced almost as soon as they are made available.

The second specific requirement of a VI feedback function, though crucial, has up to now gone unnoticed. It is generally understood that when response rate gets low enough, a VI schedule becomes functionally like continuous reinforcement (fixed ratio, or FR, 1). Each reinforcer, once available, is held until a response occurs. If the shortest time between responses exceeds the longest programmed interval, then every response will be reinforced. At the low end, therefore, the VI feedback function must approach the feedback function for continuous reinforcement, a line passing through the origin with (assuming the time units of response rate and rate of reinforcement to be the same) a slope of 1.0. This property has been represented graphically (Baum, 1973, 1981), but without recognition that it places a major mathematical constraint on the feedback function: As response rate approaches zero, the derivative of the function must approach 1.0.

Of the VI feedback functions that have been proposed, some fail both theoretically and empirically (i.e., fail to fit data), some pass theoretically but fail empirically, and some pass empirically but fail theoretically. Rachlin (1978, 1989), for example, suggested a power function, which, expressed in terms of response rate, takes the form $r = aB^m$, where m is less than 1.0. This function fails both specific theoretical requirements: (a) It lacks a horizontal asymptote—as B grows, r grows without limit. (b) It fails to approach continuous reinforcement at the low end—as B approaches zero, the derivative dr/dB approaches infinity. Finally, it fails to fit known sets of data (Nevin & Baum, 1980).

Most proposed VI feedback functions have assumed the general form

$$r = \frac{1}{t + D(B)}, \quad (1)$$

where t is the average programmed interval of the VI, and $D(B)$ is the additional time ("delay"), the average interval from the setting-up of the reinforcer to its actual delivery, a function of B , introduced by the schedule's response requirement. In words, Equation 1 states only what is true by definition: The rate of reinforcement equals the reciprocal of the obtained interval between reinforcers. With it, we can focus more specifically on the delay term D and the way it depends on B .

To arrive at possible functions $D(B)$, researchers have had to make assumptions about the temporal structure of VI responding. The earliest proposal (Baum, 1973), assuming regular responding, approximated D as one half of the average interresponse time, $0.5/B$. Staddon (1980), assuming the other extreme of perfectly random responding, generated by a Poisson process, suggested that D might equal the average interresponse time, $1/B$.

It is well known from studies of interresponse times that VI responding is neither perfectly regular nor perfectly random, but appears to embody a compromise of regularity with randomness (e.g., Blough, 1963; Schaub, 1967). Prelec and Herrnstein (1978) proposed a general form that includes perfect regularity and randomness as special cases, $D(B) = a/B$, where the parameter a might be a function of experimental conditions apart from t , such as physical characteristics of the response key or the particular intervals comprising the sched-

ule. The resultant feedback function is a hyperbola:

$$r = \frac{B}{tB + a}. \quad (2)$$

Although Equation 2 satisfies the requirement that r approach $1/t$ as B grows large, it fails to satisfy the low-end requirement with any value of a other than 1.0. The derivative of Equation 2 with B set equal to zero equals $1/a$. Because the slope of the feedback function must equal 1.0 when B equals zero, a can take no value other than 1.0. Although this verifies that Staddon's (1980) proposal passes both theoretical tests, the implied randomness of responding remains unrealistic. Not surprisingly, Equation 2 with a equal to 1.0 fails to fit known data (Nevin & Baum, 1980).

Although Prelec and Herrnstein's (1978) particular equation cannot be completely correct, nevertheless it appears to embody a point that is inescapable: To accommodate individual differences and variations in apparatus and experimental method, the feedback function needs to include one or more parameters.

Nevin and Baum (1980) derived a VI feedback function that included parameters and a compromise between random and regular responding. They assumed that VI performance consists of bursts of responding alternating with pauses. During a burst, responses were assumed to occur at a constant tempo, K . Both bursts and pauses were assumed to terminate randomly, according to two Poisson processes. Finally, the rate of terminating pauses, x , and the rate of terminating bursts, y , were assumed to vary inversely, such that the sum $x + y$ equaled a constant, c . The overall rate of reinforcement was considered to be the sum of the rate obtained during bursts and the rate obtained during pauses. The resulting equation was

$$r = \frac{B}{K} \frac{1}{t + 0.5/K} + \frac{K - B}{K} \frac{1}{t + K/cB}, \quad (3)$$

where B/K represents the proportion of time spent responding, $(K - B)/K$ (or $1 - B/K$) represents the proportion of time spent pausing, the first term represents rate of reinforcement during bursts, the second represents rate during pauses, and K/cB represents $1/x$, the average duration of a pause.

Equation 3 seemed to satisfy all the requirements. It approached $1/t$ as B grew large. It

fitted all the sets of data that Nevin and Baum examined. The derived values of K seemed to coincide with knowledge about interresponse times (e.g., Blough, 1963; Palya, 1992). When the low-end derivative rule is applied to Equation 3, however, a problem appears. The derivative with B equal to zero equals

$$r'(0) = \frac{1}{Kt + 0.5} + \frac{c}{K}.$$

When this expression is set equal to 1.0, the required value, we find that

$$c = K - \frac{K}{Kt + 0.5}. \quad (4)$$

Because the tempo K is usually a relatively large number, greater than all but the highest response rates, and t is usually considerably greater than zero, the second term on the right hand side of Equation 4 is negligible in comparison to K . The equation implies, therefore, that c must approximately equal K .

Equation 4 presents a problem for Nevin and Baum's (1980) analysis, because the values of c necessary to fit the data were always much less than K . With K equal to 100 and c equal to 10, the derivative, instead of approaching 1.0 as B approaches zero, approaches a value close to 0.1. Moreover, if c approximately equals K , Equation 2 reduces to a simple compromise between Baum's (1973) early suggestion based on regular responding and Staddon's (1980) suggestion based on random responding. Such an equation cannot fit the data.

Nevin and Baum's (1980) reasoning can be used to derive a slightly different function, more in keeping with Equation 1. Equation 3 represents a weighted average of two rates of reinforcement, that during bursts and that during pauses. Instead, the delay during a burst ($0.5/K$) can be averaged with the delay during a pause (K/cB) to produce a single expression for the delay $D(B)$:

$$r = \frac{1}{t + (B/K)(0.5/K) + (K - B)/cB}. \quad (5)$$

Although Equation 5 represents a small improvement on Equation 3 conceptually, the same linkage occurs between c and K ; when the derivative with B equal to zero— $r'(0)$ —is set equal to 1.0, one finds that c equals K .

McDowell and associates (McDowell &

Kessel, 1979; McDowell & Wixted, 1988), adopting an approach suggested by systems theory and ignoring the VI feedback function, derived a performance function relating VI responding to rate of reinforcement:

$$B = [\ln(me^{1/r} + b)]^{-1}, \quad (6)$$

where m and b are parameters and e is the base of natural logarithms. Because this fits many VI performance data, it appears to circumvent the problem of defining the feedback function.

The trouble with Equation 6 is that it assumes no feedback between responding and reinforcement; it assumes that for any given VI schedule, $1/r$ equals t , as if the VI were indistinguishable from response-independent reinforcement (a variable-time schedule). Apart from whatever other theoretical problems such an approximation may present, it is bound to be inaccurate at low response rates. As long as response rate is high relative to the programmed rate of reinforcement, the obtained rate of reinforcement approximates the programmed rate, but whenever conditions cause response rate to drop toward the programmed rate of reinforcement, rate of reinforcement becomes heavily dependent on responding, as in a ratio schedule, and drops in accordance with response rate.

McDowell and Wixted's (1988) equation for ratio schedules incorporates the linear feedback relation $1/r = n/B$, where n is the size of the ratio requirement:

$$B = [\ln(me^{n/B} + b)]^{-1}. \quad (7)$$

This is the equation that presumably would begin to apply to VI performance when response rate was low enough for feedback to be ratio-like.

McDowell's approach, if it is to work, would require some sort of compromise between effects of high and low response rates analogous to the compromise that Nevin and Baum (1980) attempted between regular and random responding. Indeed, under their assumptions the distinction between high- and low-rate performances becomes the same as the distinction between regular and random responding, because responding becomes more regular as it grows and more random as it falls. It remains to be seen whether some compromise between Equations 6 and 7 can be derived, but Equation 6 by itself cannot satisfy the requirements for a comprehensive theory of VI performance.

Strategies for Solution

The sort of approach taken by Nevin and Baum (1980) to deriving feedback functions may be called the "bottom-up" approach. Beginning with seemingly reasonable assumptions about the temporal structure of responding, one derives a function and tests it against the theoretical requirements and the data. If it fails, one goes back to the assumptions, changes them, derives a new function, and so on. Although this approach may solve the problem and need not be abandoned, it can be complemented by an alternative, the "top-down" approach.

In the top-down approach, one lists all the theoretical requirements and looks for functional forms that satisfy them. These can then be tested against the data. If one can be found that passes all tests, then it might prove possible to derive it afterwards by the bottom-up approach. The practicality of this method hinges on discovering enough possible functional forms.

Given our requirements of continuity, passing through the origin, a horizontal upper asymptote, and a slope of 1.0 at the origin, how many functional forms might be possible? The answer is uncertain. Our requirements eliminate many possibilities, however, and the ones remaining seem mostly to involve exponential terms.

In general terms, we require that as B approaches zero, the feedback function $r(B)$ should reduce to $r = B$. If we work from Equation 1, our problem becomes identifying functional forms for $D(B)$. We require that $D(B)$ become infinite as B approaches zero, that $D(B)$ approach zero or some small constant as B approaches infinity, and

$$\lim_{B \rightarrow 0} -D'(B)[t + D(B)]^{-2} = 1.$$

Because Prelec and Herrnstein's (1978) and Nevin and Baum's (1980) proposals came close to a solution, it might be helpful to consider functions of the form

$$D(B) = \frac{1}{y(B)B}, \quad (8)$$

where $y(B)$ is a function of B . Because setting y equal to a constant smaller than 1.0 ($y = 1/a$ in terms of Equation 2; $a > 1$) results in reasonable fits to data and y needs to approach 1.0 when B approaches zero, a function $y(B)$

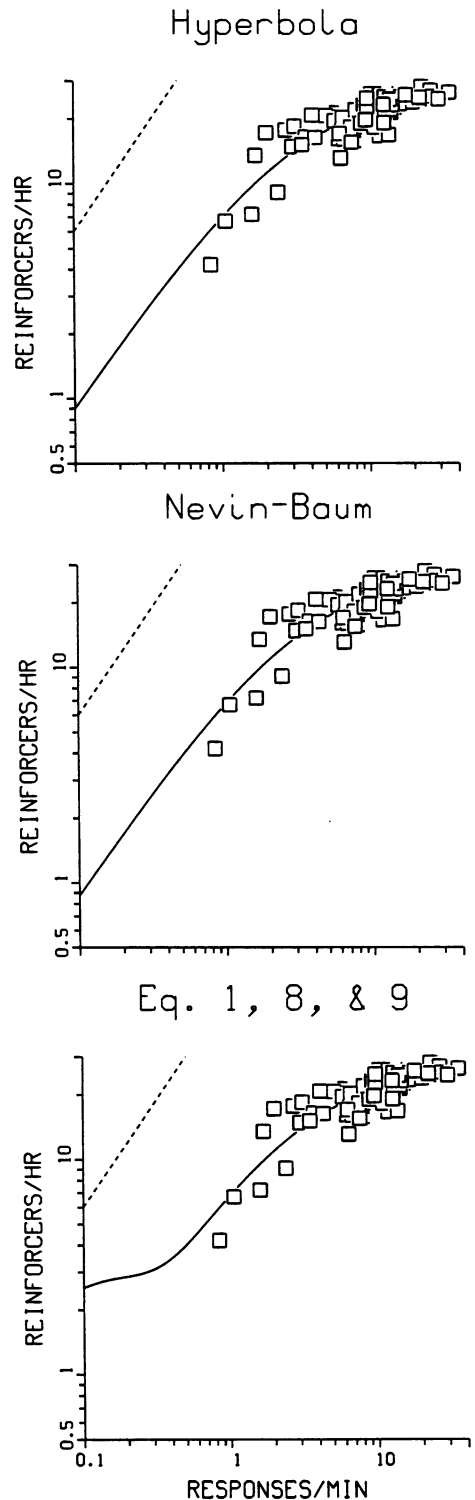


Fig. 1. The results of fitting the three models (solid curves) to performance on VI 2-min schedules in an experiment by Baum (1976). The broken line represents the feedback function for FR 1. Note logarithmic coordinates. See text for further explanation.

Table 1

Fits of the three models to Baum's (1976) data. PV: proportion of variance accounted for. S: slope of least squares regression line fitted to residuals. The parameter t gives the derived value of the VI in minutes. For explanations of the parameters K , a , and c , see text.

Model	PV	S	t	K	a	c
Hyperbola (Equation 2)	.773	0.0004	2.16	—	6.5	—
Nevin-Baum (Equation 5)	.773	0.0001	2.25	60	—	9.0
Equations 1, 8, and 9	.773	0.0001	2.14	—	0.15	10*

* Greater values produced no perceptible change in goodness of fit.

that rose to 1.0 when B approached zero and fell to approach a value less than 1.0 as B increased might offer a possible solution. For example, one function with those properties is

$$y = a + (1 - a)e^{-cB}. \tag{9}$$

This describes a curve that rises toward 1.0 as B decreases and declines asymptotically toward a as B increases. When a equals 1.0, it reduces to $y = 1$ (i.e., Staddon's, 1980, random-responding model). More parameters could be added to Equation 9, but two might suffice. The equation derived by substituting Equation 9 into Equation 8 and Equation 8 into Equation 1 satisfies all the theoretical requirements for the VI feedback function.

Fits to Data

Data sets to be fitted to possible feedback functions come from experiments in which a VI schedule was held constant across conditions while some factor other than the schedule was varied. Both the response rates and the rates of reinforcement must be reported or be derivable, and the range of variation in response rate must be wide enough to produce significant variation in rate of reinforcement. Data sets meeting all these requirements are comparatively rare. Nevin and Baum (1980) located several that can serve here. Three equations were fitted to each data set: the Prelec-Herrnstein hyperbola of Equation 2, the Nevin-Baum model as represented in Equation 5, and the equation derived by incorporating Equation 9 into Equation 8 and Equation 8 into Equation 1. They were fitted by an iterative method that minimized the sum of the squared deviations from the function in logarithmic coordinates.

Figure 1 shows the three fits to data reported by Baum (1976). Rats were exposed to concurrent schedules in which one component was always VI 2 min. Each point represents rate

of lever pressing and rate of food delivery averaged over several sessions for 1 rat; 6 rats' data were pooled. Table 1 shows the best fitting parameter values and two measures of goodness of fit: proportion of variance accounted for (PV) and the slope of a regression line fitted to the residuals (S). If this latter slope differs significantly from zero, then the data deviate systematically from the curve. All three fits appear close, with no systematic deviation. The derived values of t are close to the nominal value of 2.0. As expected from their inability to meet the low-rate theoretical requirement, the hyperbola and the Nevin-Baum equation parallel the FR 1 feedback line (broken line; $r = 60B$ here) as response rate gets low, but cannot meet it. For the hyperbola, this results from a value of a (6.5) greater than 1.0. For the Nevin-Baum equation, this results from a value of c much lower than K (9 vs. 60). The ratio of K to c approximates the value of a . The function derived from Equations 1, 8, and 9 meets the FR 1 feedback line by making a horizontal move to the left at a low range of response rates, less than one response per minute in Figure 1. The response rate at which this move begins depends on the value of c in Equation 9.

Figures 2 and 3 show the results of fitting the three equations to data from an experiment by McSweeney (1975). Pigeons' pecks were reinforced on two concurrent schedules, nominally a VI 2 min and a VI 4 min. Response rate was varied by varying deprivation. Figure 2 and Table 2 show the results for the VI 2-min schedule. Like Figure 1, Figure 2 indicates that all three equations fit the data closely. Equations 1, 8, and 9 again result in a horizontal move to the left as response rate drops below one per minute. Table 2 confirms the visual impressions. The fits are good; the derived values of t again are close to the nominal value of 2.0. Again, a is larger than 1.0 for the hyperbola, c is less than K for the

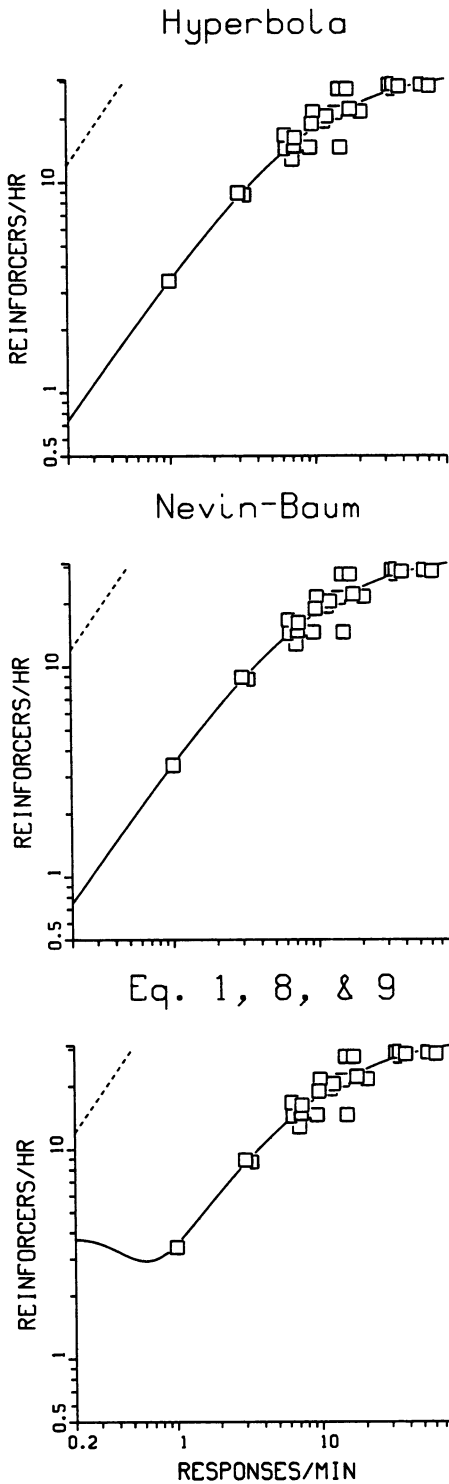


Fig. 2. The results of fitting the three models (solid curves) to performance on VI 2-min schedules in an experiment by McSweeney (1975). The broken line represents the feedback function for FR 1. Note logarithmic coordinates. See text for further explanation.

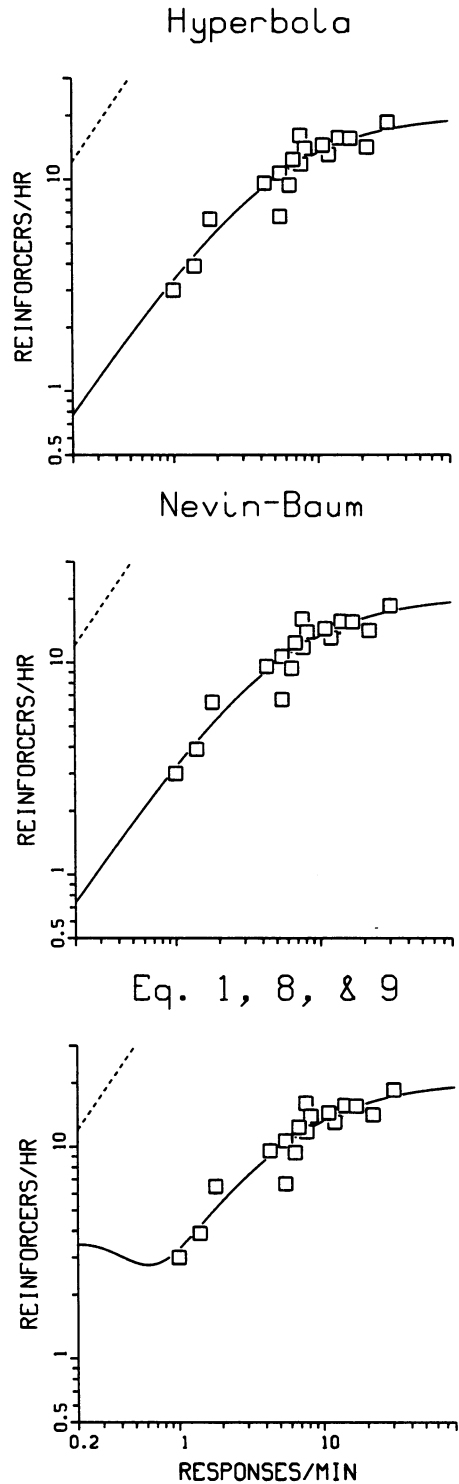


Fig. 3. The results of fitting the three models (solid curves) to performance on VI 4-min schedules in an experiment by McSweeney (1975). The broken line represents the feedback function for FR 1. Note logarithmic coordinates. See text for further explanation.

Table 2

Fits of the three models to McSweeney's (1975) data for VI 2-min schedules. PV: proportion of variance accounted for. S: slope of least squares regression line fitted to residuals. The parameter *t* gives the derived value of the VI in minutes. For explanations of the parameters *K*, *a*, and *c*, see text.

Model	PV	S	<i>t</i>	<i>K</i>	<i>a</i>	<i>c</i>
Hyperbola (Equation 2)	.928	-0.0001	1.75	—	16	—
Nevin-Baum (Equation 5)	.929	-0.0001	1.83	158	—	10
Equations 1, 8, and 9	.929	-0.0001	1.72	—	0.063	6.0

Nevin-Baum equation, and *K/c* approximately equals *a*.

Figure 3 and Table 3 show the three fits for McSweeney's VI 4-min schedule. The fits appear to be good. As before, the hyperbola and the Nevin-Baum equation fail to converge on the broken line (FR 1), and the curve generated by Equations 1, 8, and 9 moves horizontally below one response per minute to converge eventually on the line as *B* approaches zero. Table 3 shows that, as before, the fits are good and are all about equally good—proportions of variance accounted for equal about .9, and slopes fitted to residuals are close to zero. The value of *a* for the hyperbola (15.0) approximates the ratio of *K* to *c* for the Nevin-Baum equation (15.8). As Nevin and Baum (1980) noted, the derived value of *t* appears to be substantially less than the nominal value of 4.0, suggesting that the VI programmer operated differently from the way McSweeney thought.

Figure 4 shows fits of the three equations to data from an experiment by Rachlin and Baum (1972). Pigeons' pecks at a key were reinforced on a VI 3-min schedule while various schedules of response-independent food were superimposed. Each point again represents performance averaged across several sessions for 1 subject; 6 pigeons' data were pooled. Again, as *B* approaches zero, only the curve derived from Equations 1, 8, and 9 converges

on the feedback line for FR 1 (broken line). The curve accommodates the two lowest points by placing them on the horizontal segment, which again begins at about one response per minute. This was probably possible by coincidence, because the two points were generated by 2 different pigeons. Table 4 shows that accommodating the two points produced a better fit (PV = .924 vs. PV = .878) for Equations 1, 8, and 9, as would be expected, but the fits to the hyperbola and the Nevin-Baum equation were still respectable. All slopes fitted to residuals were close to zero. The derived value of *t* was close to the nominal value of 3 min. The value of *a* for the hyperbola again exceeded 1.0, and the ratio of *K* to *c* for the Nevin-Baum equation (6.74) equaled it almost exactly.

Conclusions and Questions

Each of the three models considered has its advantages and disadvantages. The great advantage of the hyperbola is its simplicity. Because it seems to fit data with values of *a* between 6 and 16, it might be useful as an approximation to the VI feedback function for the purpose of constructing theoretical accounts of operant performance (i.e., combining feedback functions with functional relations; e.g., Baum, 1981). The correspondence between the values of *a* (Tables 1 through 4)

Table 3

Fits of the three models to McSweeney's (1975) data for VI 4-min schedules. PV: proportion of variance accounted for. S: slope of least squares regression line fitted to residuals. The parameter *t* gives the derived value of the VI in minutes. For explanations of the parameters *K*, *a*, and *c*, see text.

Model	PV	S	<i>t</i>	<i>K</i>	<i>a</i>	<i>c</i>
Hyperbola (Equation 2)	.899	0.0008	3.00	—	15	—
Nevin-Baum (Equation 5)	.900	0.0000	3.00	178	—	11.3
Equations 1, 8, and 9	.899	0.0002	2.94	—	0.063	6 ^a

^a Greater values produced no perceptible change in goodness of fit.

and the ratio K/c suggests that the hyperbola might be a good approximation to the Nevin-Baum equation (Equation 5). The second term in the denominator of Equation 5, $0.5B/K^2$, takes on only small values; even at its maximum, when B equals K , the quantity $0.5/K$ is normally a small number. We can consider that term negligible and focus on the third term, $(K - B)/cB$. If we divide the top and bottom of the hyperbola (Equation 2) by B , the corresponding term there is a/B . When B is small, in the range of lower response rates that determines the value of a , K is large enough relative to B that the term $(K - B)/cB$ approximately equals K/cB . This explains the correspondence between a and K/c and supports the usefulness of the hyperbola as an approximation to the Nevin-Baum equation.

The strength of the Nevin-Baum equation (Equation 5) is that it stems from the bottom-up approach—that is, it flows from plausible propositions about the structure of operant behavior. The trouble with the hyperbola and Equations 8 and 9 is the trouble with the top-down method: The equations have no rationale other than that they work. One would hope for a feedback function that reflected verifiable properties of the underlying performance. The virtue of the bottom-up approach, as exemplified by Nevin and Baum (1980), is that the assumptions can, at least in principle, be tested and tied into the body of knowledge about operant behavior. The assumption that operant behavior occurs in alternating bursts and pauses can be tested against data; if true, it presents an organizing principle that might be useful in many contexts, not just in deriving feedback functions.

The advantage to Equations 8 and 9 over the hyperbola and the Nevin-Baum equation is that Equations 8 and 9 result in a feedback function that both fits the data and is theoretically correct. If one could, using reasonable assumptions in a bottom-up approach, derive a function similar in form to this feedback function, satisfying the low-rate approach to the FR 1 feedback line, the result might be truly illuminating.

One might still maintain that the Nevin-Baum model and the hyperbola work well enough for most purposes and that pursuit of fine theoretical points might be a waste of time. A partial answer is that the pursuit of theoretical correctness leads to interesting empirical questions.

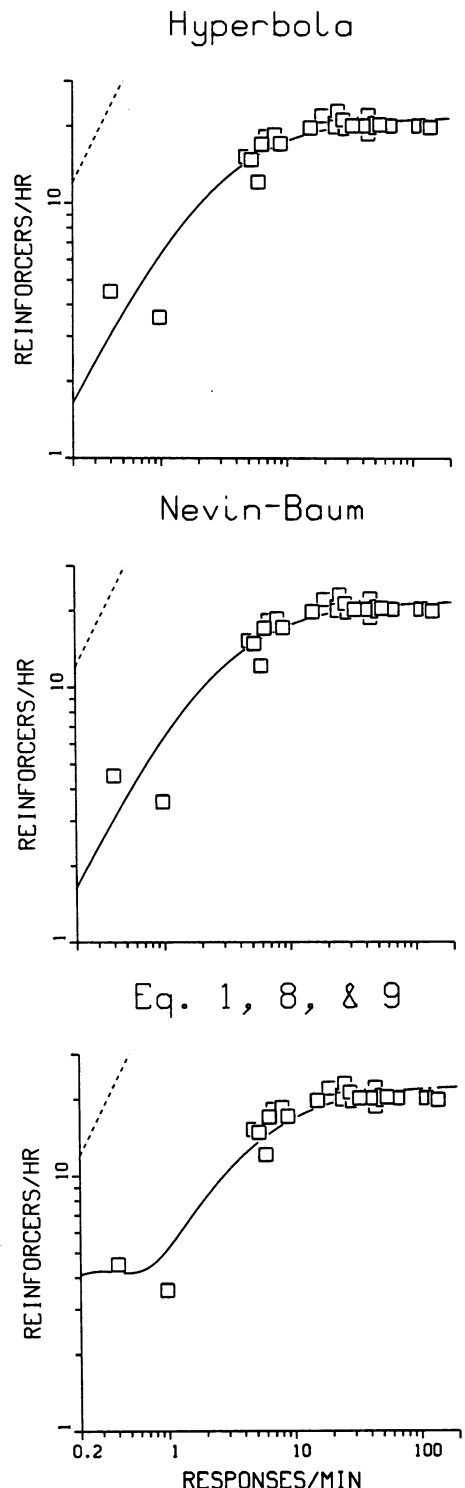


Fig. 4. The results of fitting the three models (solid curves) to performance on VI 3-min schedules in an experiment by Rachlin and Baum (1972). The broken line represents the feedback function for FR 1. Note logarithmic coordinates. See text for further explanation.

Table 4

Fits of the three models to Rachlin and Baum's (1972) data. PV: proportion of variance accounted for. S: slope of least squares regression line fitted to residuals. The parameter t gives the derived value of the VI in minutes. For explanations of the parameters K , a , and c , see text.

Model	PV	S	t	K	a	c
Hyperbola (Equation 2)	.878	-0.0003	2.77	—	6.8	—
Nevin-Baum (Equation 5)	.878	-0.0003	2.81	159	—	23.6
Equations 1, 8, and 9	.924	-0.0005	2.66	—	0.11	5.31

The very success of the theoretically incorrect Equations 2 and 5 raises a puzzle. The low response rates in Figures 1, 2, 3, and 4 produce remarkably low rates of reinforcement. Although responses occurring randomly at a rate of one per minute on a VI 2-min schedule would produce fewer than 30 reinforcers per hour, they could still be expected to produce an average of 20 reinforcers per hour [$60/(2 + 1)$]. Inspection of Figures 1 and 2 reveals that the rates actually produced were much lower—about four reinforcers per hour. Similar remarks apply to Figure 3 (VI 4 min) and Figure 4 (VI 3 min).

Alternatively, one could look at the low rates of reinforcement as paired with response rates that are too high. Rates of reinforcement on the order of four per hour would go with extremely low response rates if each response occurred in isolation. If the response rates are anomalously high, it might be because the responses occur in clusters or bursts (separated, of course, by long pauses). The value of the hyperbola's parameter a (or K/c for the Nevin-Baum equation) reflects the average number of responses in such a burst. If, after a long pause, six or seven responses (Table 1) occurred in a very brief time interval, only one of the responses would be likely to be reinforced. Although usually that response would be the first, under some circumstances, such as the presence of a changeover delay (COD), it could be a later response.

The use of a COD might have contributed to clustered responding in Baum's (1976) experiment and McSweeney's (1975) experiment. If no responding occurred on one alternative for a long time, and the first response on the alternative initiated a COD during which reinforcement was prevented, several responses (15 or 16 in McSweeney's experiment, six or seven in Baum's; Tables 1, 2, and 3) might occur before the COD ended and a

response produced a reinforcer. With long times spent at the other alternative, every changeover to the nonpreferred alternative might lead to a reinforcer, but the low rate of reinforcement there would accompany an anomalously high response rate because of all the responses made during the COD.

Such an explanation, however, would fail with the Rachlin-Baum (1972) experiment, because the alternative reinforcement there was presented independently of responding. Because there was no feature comparable to a COD, the first response after the VI timed out produced a reinforcer. If responses occurred in bursts with only one reinforcer per burst, then each burst (about seven responses; Table 4) must have followed a reinforcer. This would result in a pattern of a long pause, followed by a single response and reinforcer, followed by a burst of responses, followed again by a pause, and so on.

Future research might pursue both the theoretical and the empirical problems raised. Can we derive a theoretically correct feedback function from basic propositions about operant performance—a bottom-up approach? What patterns of responding develop when response rate gets low? Some experiments in which response rate is varied over a wide range for simple VI schedules might be especially helpful.

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